

# The cosmological ${}^7\text{Li}$ problem from a nuclear physics perspective

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## Abstract

The primordial abundance of  ${}^7\text{Li}$  as predicted by Big Bang Nucleosynthesis (BBN) is more than a factor 2 larger than what has been observed in metal-poor halo stars. Herein, we analyze the possibility that this discrepancy originates from incorrect assumptions about the nuclear reaction cross sections relevant for BBN. To do this, we introduce an efficient method to calculate the changes in the  ${}^7\text{Li}$  abundance produced by arbitrary (temperature dependent) modifications of the nuclear reaction rates. Then, considering that  ${}^7\text{Li}$  is mainly produced from  ${}^7\text{Be}$  via the electron capture process  ${}^7\text{Be} + e^- \rightarrow {}^7\text{Li} + \nu_e$ , we assess the impact of the various channels of  ${}^7\text{Be}$  destruction. Differently from previous analysis, we consider the role of unknown resonances by using a complete formalism which takes into account the effect of Coulomb and centrifugal barrier penetration and that does not rely on the use of the narrow resonance approximation. As a result of this, the parameter space for a nuclear physics solution of the  ${}^7\text{Li}$  problem is considerably reduced. We exclude that resonant destruction in the channels  ${}^7\text{Be} + t$  and  ${}^7\text{Be} + {}^3\text{He}$  can explain the  ${}^7\text{Li}$  puzzle. Resonances in  ${}^7\text{Be} + d$  and  ${}^7\text{Be} + \alpha$  could potentially produce relevant effects but very favorable conditions are required. For the  ${}^7\text{Be} + \alpha$  channel, the possibility of a (partially) suitable resonant level in  ${}^{11}\text{C}$  is studied in the framework of a coupled-channel model.

# 1 Introduction

Big Bang Nucleosynthesis (BBN) is one of the solid pillars of the standard cosmological model and represents the earliest event in the history of the universe for which confirmable predictions can be made (see [1] for a review). The theory predicts that relevant abundances of light elements, namely  $^2\text{H}$ ,  $^3\text{He}$ ,  $^4\text{He}$  and  $^7\text{Li}$ , were produced during the first minutes of the evolution of the universe. Theoretical calculations of these abundances are well defined and are very precise. The largest uncertainties arise from the values of cross-sections of the relevant nuclear reactions and are at the level of 0.2% for  $^4\text{He}$ , 5% for  $^2\text{H}$  and  $^3\text{He}$  and 15% for  $^7\text{Li}$ <sup>1</sup>.

In standard BBN, the primordial abundances depend on only one free parameter, the present baryon-to-photon ratio  $\eta \equiv (N_{\text{B}} - N_{\bar{\text{B}}})/N_{\gamma}$ , which is related to the baryon density of the universe by  $\Omega_{\text{B}}h^2 = 3.65 \cdot 10^7 \eta$ . This quantity can be constrained with high accuracy from the observation of the anisotropies of the Cosmic Microwave Background (CMB). The latest WMAP-7 results suggest  $\Omega_{\text{B}}h^2 = 0.02249 \pm 0.00056$ , which corresponds to  $\eta_{\text{CMB}} = 6.16 \pm 0.15 \times 10^{-10}$  [5]. If this value is accepted, then BBN is a parameter free theory which can be used to test the standard cosmological model and/or the chemical evolution of the universe.

Comparison of theoretical predictions with observational data is not straightforward. Data are subject to poorly known evolutionary effects and there are systematic errors. Even so, the agreement between the predicted primordial abundances of  $^2\text{H}$  and  $^4\text{He}$  and the values inferred from observations is non-trivial. However, the situation is much more complicated for  $^7\text{Li}$ . Using  $\eta = \eta_{\text{CMB}}$ , the predicted primordial  $^7\text{Li}$  abundance is [6]

$$(\text{Li}/\text{H})_{\text{BBN}} \simeq (5.1_{-0.6}^{+0.7}) \times 10^{-10}. \quad (1)$$

This is a factor  $\sim 3$  larger than that inferred by observing the so-called ‘Spite Plateau’ in the  $^7\text{Li}$  abundance of metal-poor halo stars, which has been given [1] as

$$(\text{Li}/\text{H})_{\text{obs}} \simeq (1.7 \pm 0.06 \pm 0.44) \times 10^{-10}. \quad (2)$$

The quoted errors take into account the dispersion of the various observational determinations. Moreover, it is considered that Lithium in Pop II stars can be destroyed as a consequence of mixing of the outer layers with the hotter interior. This process can be constrained by the absence of significant scatter in Li versus Fe in the Spite Plateau [8].

The abundance of  $^7\text{Li}$  is a central unresolved issue in BBN [6] about which there has been recent concern [7, 8, 9] regarding erroneous evaluation of nuclear reaction rates responsible for  $^7\text{Li}$  production. At  $\eta = \eta_{\text{CMB}}$ ,  $^7\text{Li}$  is mainly produced from  $^7\text{Be}$  via the electron capture process  $e^- + ^7\text{Be} \rightarrow ^7\text{Li} + \nu_e$ . Thus nuclear reactions producing and destroying  $^7\text{Be}$  must be considered. The leading processes,  $^3\text{He}(\alpha, \gamma)^7\text{Be}$  and  $^7\text{Be}(n, p)^7\text{Li}$ , have been well studied and the cross sections are known to a few percent accuracy [3]. In [10], it was noted that an increase by a factor greater than 1000 in the sub-dominant  $^7\text{Be}(d, p)2\alpha$  cross section could provide the necessary suppression of  $^7\text{Li}$ . This enhancement was not found in experimental data [11] but could have escaped detection if it were produced by a sufficiently narrow resonance, as suggested in [12]. Other possible resonant destruction channels have been considered [13] as well, such as the channels  $^7\text{Be} + ^3\text{He} \rightarrow ^{10}\text{C}$  and  $^7\text{Be} + t \rightarrow ^{10}\text{B}$  that await experimental verification.

In this paper, we consider further the cosmological  $^7\text{Li}$  problem from the nuclear physics perspective. To do this, in Sec. 2, we introduce an efficient method to calculate the response of the  $^7\text{Li}$  primordial abundance to an arbitrary modification of the nuclear reaction rates. This approach leads to an understanding, in simple physical terms, of why it is so difficult to find a nuclear physics solution to the observed discrepancy. Then, in Secs. 4 and 5, the various possible  $^7\text{Be}$  destruction channels are considered, including possible new resonances.

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<sup>1</sup> After the first evaluations [2], theoretical uncertainties in BBN have been carefully assessed in several papers. Monte-Carlo and semi-analytical approaches have been used. As examples see Refs. [3, 4].

This has been suggested recently also in Ref. [13], but here we use a more refined description of the nuclear processes than in previous investigations. In particular, we do not assume the narrow resonance approximation and we include the effect of the Coulomb and centrifugal barrier penetration in our parametrisation of resonating cross sections. As a result, the parameter space for a nuclear physics solution of the  ${}^7\text{Li}$  puzzle is considerably reduced. Our conclusions are summarised in Sec.6.

## 2 The ${}^7\text{Li}$ response to nuclear reaction rate modifications

It is useful to briefly review the light element production mechanism in the early universe. The abundance of a generic element  $i$  in the early universe,  $Y_i = n_i/n_B$  where  $n_B$  is the baryon number density, evolves according to the rate equations,

$$\frac{dY_i}{dt} = n_B \left[ \sum_{j,k} Y_j Y_k \langle \sigma_{jk} v \rangle_T - \sum_l Y_i Y_l \langle \sigma_{il} v \rangle_T \right]. \quad (3)$$

The sums include the relevant production and destruction reactions and  $\langle \sigma_{ij} v \rangle_T$  are the thermally averaged cross sections. It is known [14] that a good approximation is obtained by studying the quasi-fixed point of the above equations, *viz.*

$$Y_i \sim \left. \frac{C_i}{D_i} \right|_{T=T_{i,f}}, \quad (4)$$

where  $C_i$  and  $D_i$  are the total rate of creation and destruction of the  $i$ -element, given by

$$C_i = n_B \sum_{j,k} Y_j Y_k \langle \sigma_{jk} v \rangle_T, \quad (5)$$

and

$$D_i = n_B \sum_l Y_l \langle \sigma_{il} v \rangle_T. \quad (6)$$

$T_{i,f}$  is the freeze-out temperature for the  $i$ -element, namely the temperature below which the rates  $C_i$  and  $D_i$  become smaller than the Hubble expansion rate (see Ref. [14] for details).

At  $\eta = \eta_{\text{CMB}} \simeq 6 \times 10^{-10}$ ,  ${}^7\text{Li}$  is mainly produced from  ${}^7\text{Be}$  that undergoes at late times (*i.e.* long after the  ${}^7\text{Be}$  freeze-out) the electron capture process  $e^- + {}^7\text{Be} \rightarrow {}^7\text{Li} + \nu_e$ , so that we have

$$Y_{\text{Li}} \sim Y_{\text{Be}} \sim \left. \frac{C_{\text{Be}}}{D_{\text{Be}}} \right|_{T=T_{\text{Be,f}}}, \quad (7)$$

where  $C_{\text{Be}}$  and  $D_{\text{Be}}$  are the total  ${}^7\text{Be}$  production and destruction rates. The dominant  ${}^7\text{Be}$  production mechanism is through the capture reaction,  ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$ ; a reaction that has been studied in detail both experimentally [15] and theoretically [16]. The cross section is known to  $\sim 3\%$  uncertainty. The dominant  ${}^7\text{Be}$  destruction channel is the process  ${}^7\text{Be}(n, p){}^7\text{Li}$ ; the cross section of which now is known to  $\sim 1\%$  accuracy as we discuss in the next section. As these leading processes have been well studied, a sizeable reduction of the  ${}^7\text{Li}$  predicted abundance occurs only if large increases of the sub-dominant  ${}^7\text{Be}$  destruction channels are allowed as may be the case if new, so far unknown, resonances become influential. To study this possibility, we introduce a simple formalism to describe the response of  ${}^7\text{Li}$  to a generic (temperature dependent) modification of the nuclear reaction rates. Motivated by Eq.(7), we assume that a linear relation exists between the *inverse*  ${}^7\text{Li}$  abundance,  $X_{\text{Li}} \equiv 1/Y_{\text{Li}}$ , and the total  ${}^7\text{Be}$  destruction rate. This relation can be expressed in general terms as<sup>2</sup>,

$$\delta X_{\text{Li}} = \int \frac{dT}{T} K(T) \delta D_{\text{Be}}(T), \quad (8)$$

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<sup>2</sup>Here and in the following, the notation  $\overline{Q}$  refers to the standard value for the generic quantity  $Q$ .

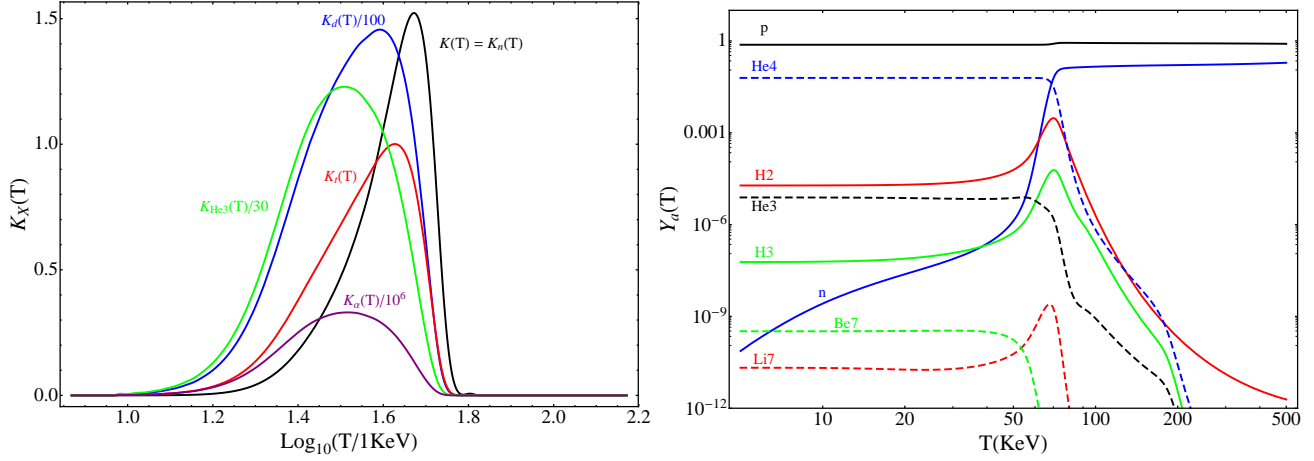


Figure 1: *Left Panel: The kernels  $K_a(T)$  defined in Eqs.(8,14). Right Panel: The evolution of light element abundances  $Y_i = n_i/n_B$  as a function of temperature, assuming  $\eta = \eta_{\text{CMB}}$  in standard BBN.*

where

$$\delta X_{\text{Li}} = \frac{X_{\text{Li}}}{\bar{X}_{\text{Li}}} - 1, \quad (9)$$

and

$$\delta D_{\text{Be}}(T) = \frac{D_{\text{Be}}(T)}{\bar{D}_{\text{Be}}(T)} - 1. \quad (10)$$

The integral kernel  $K(T)$  has been evaluated numerically by considering the effects of localised (in temperature) increases of the reaction rate  $D_{\text{Be}}(T)$ . Our results are shown by the black solid line in the left panel of Fig. 1. We checked numerically that Eq. (8) describes accurately large variations of the  ${}^7\text{Li}$  abundance (up to a factor  $\sim 2$ ) and, thus, it is adequate for our purposes. The kernel  $K(T)$  is peaked at  $\sim 50$  keV; roughly corresponding to the  ${}^7\text{Be}$  freeze-out temperature  $T_{\text{Be,f}}$ . The area under the curve is equal to  $\approx 0.7$  which indicates that the total destruction rate of  ${}^7\text{Be}$  should be increased by a factor  $\sim 2.5$  to obtain a factor 2 reduction in the abundance of  ${}^7\text{Li}$ .

We can use Eq. (8) to assess the sensitivity of the abundance of  ${}^7\text{Li}$  with respect to a specific reaction channel. The total  ${}^7\text{Be}$  reaction rate is given by

$$D_{\text{Be}}(T) = n_B \sum_a Y_a(T) \langle \sigma_a v \rangle_T, \quad (11)$$

where  $\sigma_a$  is the cross section of the reaction  ${}^7\text{Be} + a$  and  $Y_a$  represents the elemental abundance of the  $a$  nuclei. In standard BBN, the dominant contribution is provided by the  ${}^7\text{Be}(n, p){}^7\text{Li}$  reaction; accounting for about  $\sim 97\%$  of the total  ${}^7\text{Be}$  destruction rate. The standard rate  $\bar{D}_{\text{Be}}(T)$  can then be set with a few percent accuracy by,

$$\bar{D}_{\text{Be}}(T) \simeq n_B \bar{Y}_n(T) \langle \bar{\sigma}_{\text{np}} v \rangle_T, \quad (12)$$

where  $Y_n(T)$  is the neutron abundance and  $\sigma_{\text{np}}$  is the cross section of  ${}^7\text{Be}(n, p){}^7\text{Li}$ . Sub-dominant reaction channels can provide a non-negligible contribution only if there is a large increase of their assumed cross section values. The fractional enhancement of  $D_{\text{Be}}(T)$  due to a generic  ${}^7\text{Be} + a$  process can be evaluated from,

$$\delta D_{\text{Be},a}(T) = \frac{\bar{Y}_a(T)}{\bar{Y}_n(T)} \frac{\langle \sigma_a v \rangle_T}{\langle \bar{\sigma}_{\text{np}} v \rangle_T}, \quad (13)$$

under the reasonable assumption<sup>3</sup> that the inclusion of a new channel for  ${}^7\text{Be}$  destruction does not alter the abundance of the  $a$  nuclei. In the right panel of Fig. 1, we show the light element abundances,  $\bar{Y}_a(T)$ , calculated assuming  $\eta = \eta_{\text{CMB}}$  in standard BBN. We see that  $d$ ,  ${}^3\text{He}$  and  ${}^4\text{He}$  have abundances larger or comparable to that of neutrons at the temperature  $T \sim 50$  keV relevant for  ${}^7\text{Be}$  synthesis. Thus reactions involving these nuclei could provide a non-negligible contribution to  $D_{\text{Be}}(T)$  even if their cross sections are lower than that of  ${}^7\text{Be}(n, p){}^7\text{Li}$ . This point can be expressed quantitatively by rewriting Eq. (8) as

$$\delta X_{\text{Li}} = \sum_a \int \frac{dT}{T} K_a(T) \frac{\langle \sigma_a v \rangle_T}{\langle \bar{\sigma}_{\text{np}} v \rangle_T}, \quad (14)$$

where

$$K_a(T) = K(T) \frac{\bar{Y}_a(T)}{\bar{Y}_n(T)}. \quad (15)$$

The kernels  $K_a(T)$  are shown in the left panel of Fig. 1 for the cases  $a = n, d, t, {}^3\text{He}, {}^4\text{He}$  and can be used to quantify the requirements for a nuclear physics solution of the cosmic  ${}^7\text{Li}$  problem. To be more consistent with observed data, a reduction of the  ${}^7\text{Li}$  abundance by a factor 2 or more is required and that corresponds to  $\delta X_{\text{Li}} \geq 1$ . To obtain this, the ratios  $R_a \equiv \langle \sigma_a v \rangle_T / \langle \bar{\sigma}_{\text{np}} v \rangle_T$  at temperatures  $T \simeq 10 - 60$  keV should be,  $R_n \geq 1.5$  for additional reactions in the  ${}^7\text{Be} + n$  channel,  $R_d \geq 0.01$  for reactions in the  ${}^7\text{Be} + d$  channel,  $R_t \geq 1.5$  for reactions in the  ${}^7\text{Be} + t$  channel,  $R_{\text{He3}} \geq 0.03$  for reactions in the  ${}^7\text{Be} + {}^3\text{He}$  channel, and  $R_{\text{He4}} \geq 4 \times 10^{-6}$  for reactions in the  ${}^7\text{Be} + {}^4\text{He}$  channel. In the next section, we explore these possibilities on the basis of general nuclear physics arguments.

### 3 Treatment of nuclear cross-sections

Except for spin statistical factors, the partial reaction cross section for a generic process,  ${}^7\text{Be} + a$ , cannot be larger than  $\sigma_{\text{max}} = (2l + 1) \pi \lambda^2$  where  $l$  is the orbital angular momentum of the channel considered,  $\lambda = 1/k$ , and  $k$  is the momentum in the center of mass (CM). The relation can be rewritten as

$$\sigma_{\text{max}} = (2l + 1) \pi \lambda^2 = (2l + 1) \frac{\pi}{2\mu E} \quad (16)$$

where  $E$  is the CM energy and  $\mu$  is the reduced mass of the colliding nuclei.

For low-energy reactions involving charged nuclei and/or a non-vanishing angular momentum, the cross section is suppressed due quantum tunnelling through the Coulomb and/or centrifugal barrier. Modelling the nuclear interaction potential by a square well with a radius  $R$ , the partial cross section for the formation of a compound system  $C$  in the process  ${}^7\text{Be} + a \rightarrow (C)$  can be expressed as,

$$\sigma_C = \sigma_{\text{max}} T_l. \quad (17)$$

The factor  $T_l$  represents the transmission coefficient for the channel considered. In the low-energy limit,  $k \ll K$ , it can be calculated from,

$$T_l = \frac{4k}{K} v_l. \quad (18)$$

$K$  is the relative momentum of the particles inside the compound system. The functions  $v_l$  are known as penetration factors [17].

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<sup>3</sup> As we see in the right panel of Fig. 1, the abundance of Beryllium is much lower than the abundances of  $d$ ,  $t$ ,  ${}^3\text{He}$  and  ${}^4\text{He}$ . This implies that a tiny fraction of these elements can potentially produce a very large depletion of Beryllium nuclei.

For uncharged particles, the penetration factors  $v_l$  are given by

$$v_l \equiv \frac{1}{G_l^2(R) + F_l^2(R)} \quad (19)$$

where  $F_l(R)$  and  $G_l(R)$  are the regular and irregular solutions of the Schrödinger radial equation, which have been tabulated in Ref. [18]. Feshbach and Lax in 1948 also tabulated the associated functions  $v'_l$ . For the lowest angular momenta,

$$\begin{aligned} v_0 &= 1 & v'_0 &= 1 \\ v_1 &= \frac{x^2}{1+x^2} & v'_1 &= \frac{1}{x^2} + \left(1 - \frac{1}{x^2}\right)^2, \end{aligned}$$

where  $x \equiv kR = \sqrt{2\mu E} R$ . The transmission coefficient for reactions involving neutrons can be calculated exactly. One obtains

$$T_l = \frac{4xXv_l}{X^2 + (2xX + x^2v'_l)v_l}, \quad (20)$$

where  $X \equiv KR$ . Eq. (20) coincides with Eq. (18) in the low-energy limit.

For charged nuclei, one has to rely on numerical calculations. An approximate expression for the penetration factors  $v_l$  can be obtained by using the WKB approximation. For the collision energy  $E$  lower than the height of the potential barrier,

$$v_l = \frac{k_l(R)}{k} \exp \left[ -2 \int_R^{r_0} k_l(r) dr \right], \quad (21)$$

where  $r_0$  is the classical distance of closest approach while  $k_l(r)$  is given by

$$k_l(r) = \sqrt{2\mu U_l(r) - k^2}, \quad (22)$$

with

$$U_l(r) = \frac{Z_a Z_X e^2}{r} + \frac{l(l+1)}{2\mu r^2}. \quad (23)$$

Eq. (21), however, does not produce accurate results when the collision energy is close to the height of the potential barrier and then exact expressions for the penetration factors  $v_l$  have to be used. When the Coulomb interaction is taken into account, Eq. (19) is still valid, but now  $F_l(R)$  and  $G_l(R)$  are Coulomb Functions, namely the regular and irregular solutions of the Schrödinger radial equation that include the Coulomb potential. Such functions can be evaluated numerically with standard numerical techniques.

In the presence of an isolated resonance, the cross section for a generic process  ${}^7\text{Be} + a \rightarrow C^* \rightarrow b + Y$  can be described by the Breit-Wigner formula,

$$\sigma = \frac{\pi \omega}{2\mu E} \frac{\Gamma_{\text{in}} \Gamma_{\text{out}}}{(E - E_r)^2 + \Gamma_{\text{tot}}^2/4}, \quad (24)$$

where  $E_r$  is the resonance energy,  $\Gamma_{\text{in}}$  is the width of the entrance channel,  $\Gamma_{\text{out}}$  is the width for the exit channel and  $\Gamma_{\text{tot}} = \Gamma_{\text{in}} + \Gamma_{\text{out}} + \dots$  is the total resonance width. The first factor in the right hand side of Eq. (24) is an upper limit for the cross section. Basically it coincides with  $\sigma_{\text{max}}$  apart from the factor,

$$\omega = \frac{2J_{C^*} + 1}{(2J_a + 1)(2J_7 + 1)}, \quad (25)$$

that takes into account the angular momenta  $J_a$  and  $J_7$  of the colliding nuclei and the angular momentum  $J_{C^*}$  of the excited state in the compound nucleus. The resonance width  $\Gamma_{\text{in}}$  (and  $\Gamma_{\text{out}}$ ) can be expressed as the product,

$$\Gamma_{\text{in}} = 2P_l(E, R) \gamma_{\text{in}}^2, \quad (26)$$

where the functions  $P_l(E, R)$  describe the Coulomb and centrifugal barrier penetrations and are related to the penetration factors  $\nu_l$  by<sup>4</sup>,

$$P_l(E, R) \equiv kR \nu_l. \quad (27)$$

The reduced width,  $\gamma_{\text{in}}^2$ , incorporates all the unknown properties of the nuclear interior. This has to be smaller than the Wigner limiting width,  $\gamma_{\text{W}}^2$  that is given by [20],

$$\gamma_{\text{in}}^2 \leq \gamma_{\text{W}}^2 = \frac{3}{2\mu R^2}. \quad (28)$$

## 4 The ${}^7\text{Be} + n$ channel

At  $T \sim T_{\text{Be,f}} \simeq 50 \text{ keV}$ , the  ${}^7\text{Be}(n, p){}^7\text{Li}$  reaction accounts for  $\sim 97\%$  of the total  ${}^7\text{Be}$  destruction rate. This process has been very well studied. Experimental data have been obtained either from direct measurements or from the  ${}^7\text{Li}(p, n){}^7\text{Be}$  inverse reaction (see Ref. [21] and references contained therein for details). The cross section for this reaction near threshold is strongly enhanced by a  $2^-$  resonance at  $E_x = 18.91 \text{ MeV}$ <sup>5</sup>. In addition, the data show evidence for two peaks that correspond to the (unresolved)  $3^+$  states at 19.07 and 19.24 MeV and to the  $3^+$  resonance at 21.5 MeV. The reaction rate has been determined by  $R$ -matrix fits to the experimental data with uncertainties  $\sim 1\%$  [3, 21]. More refined theoretical approaches based on modern coupled-channel effective field theory lead essentially to the same conclusion [22].

The cross section of  ${}^7\text{Be}(n, p){}^7\text{Li}$  reaction is extremely large. The Maxwellian-averaged cross section at thermal energies  $\langle \sigma_{\text{np}} \rangle_T = 3.84 \times 10^4 \text{ barn}$  [23, 24] is the largest thermal cross section known in the light element region. At the relevant energies for  ${}^7\text{Be}$  synthesis,  $E_{\text{Be}} \simeq T_{\text{Be,f}} \simeq 50 \text{ keV}$ , we have  $\sigma_{\text{np}}(E_{\text{Be}}) \simeq 9 \text{ barn}$ ; a value quite close to the unitarity bound value of  $\sigma_{\text{max}}(E_{\text{Be}}) = \pi/(2\mu E_{\text{Be}}) \simeq 15 \text{ barn}$ . Therein  $\mu$  is the reduced mass of the  ${}^7\text{Be} + n$  system. The size of this cross section makes it difficult to find comparable channels for  ${}^7\text{Be}$  destruction.

As an alternative process, we consider the reaction  ${}^7\text{Be}(n, \alpha){}^4\text{He}$  for which no experimental data exist in the energy range relevant for primordial nucleosynthesis. This process normally is included in BBN calculations by using the old reaction rate estimate [25] to which is conservatively assigned a factor 10 uncertainty [3]. The process  ${}^7\text{Be}(n, \alpha){}^4\text{He}$  is the second most important contribution to the  ${}^7\text{Be}$  rate of destruction, accounting for  $\sim 2.5\%$  of the total. Nevertheless, due to the large uncertainty assigned, it provides the dominant contribution to theoretical errors in the  ${}^7\text{Li}$  abundance evaluations [3].

To obtain a factor of 2 reduction in the cosmic  ${}^7\text{Li}$  abundance, the cross section of the  ${}^7\text{Be}(n, \alpha){}^4\text{He}$  reaction need be increased  $\sim 60$ -fold to have  $\sigma_{\text{n}\alpha}(E_{\text{Be}}) \simeq 1.5 \sigma_{\text{np}}(E_{\text{Be}}) \sim 15 \text{ barn}$ ; an extremely unlikely possibility. An upper bound on the non-resonant contribution to  $\sigma_{\text{n}\alpha}$  can be obtained by considering the upper limit on the Maxwellian-averaged cross section  $\langle \sigma_{\text{n}\alpha} \rangle_T \leq 0.1 \text{ mbarn}$  that was derived [26] using thermal neutrons. Due to parity conservation of strong interactions, the process cannot proceed via an  $s$ -wave collision. If we assume a  $p$ -wave collision, the measured value can be rescaled to  $T_{\text{Be,f}} \sim 50 \text{ keV}$  according to  $\langle \sigma_{\text{n}\alpha} \rangle_T \propto \sqrt{T}$ , obtaining the value  $\langle \sigma_{\text{n}\alpha} \rangle_{T_{\text{Be,f}}} \leq 0.02 \langle \sigma_{\text{np}} \rangle_{T_{\text{Be,f}}}$ . That result is much lower than what is required to solve the  ${}^7\text{Li}$  problem.

<sup>4</sup>Note that in Ref. [19], the quantities  $P_l(E, R)$  themselves are referred to as the penetration factors.

<sup>5</sup> The entrance energy of the  ${}^7\text{Be} + n$  channel with respect to the  ${}^8\text{Be}$  ground state is  $E_{\text{in}} = 18.8997 \text{ MeV}$ . Thus the listed resonances correspond to a collision kinetic energy equal to  $E_r = E_x - E_{\text{in}} = 0.01, 0.17, 0.34$  and  $2.6 \text{ MeV}$ , respectively.

One can question this estimate because it involves extrapolation over several orders of magnitude. Irrespective of this, the process  ${}^7\text{Be}(n, \alpha){}^4\text{He}$  will be suppressed at low energies with respect to  ${}^7\text{Be}(n, p){}^7\text{Li}$  because of centrifugal barrier penetration; a quantitative estimate of which can be obtained by using the results discussed in the previous section. At low energy,  $\sigma_{n\alpha}/\sigma_{np} \sim T_1/T_0 \sim 2\mu ER^2$  where  $T_0$  ( $T_1$ ) is the transmission coefficient for an  $s$ -wave ( $p$ -wave) collision in the  ${}^7\text{Be} + n$  entrance channel. By considering  $E = E_{\text{Be}}$ , and by taking  $R \leq 10$  fm as a conservative upper limit for the entrance channel radius, we obtain  $\sigma_{n\alpha}(E_{\text{Be}}) \leq 0.2 \sigma_{np}(E_{\text{Be}})$ ; that is also insufficient to explain the  ${}^7\text{Li}$  discrepancy.

Finally, we note that we do not expect a large resonant contribution to the  ${}^7\text{Be}(n, \alpha){}^4\text{He}$  cross section. The  ${}^8\text{Be}$  excited states relevant for  ${}^7\text{Be}(n, p){}^7\text{Li}$  reaction, due to parity conservation, do not decay by  $\alpha$ -emission. In summary, in view of experimental and theoretical considerations, it appears unlikely that the  ${}^7\text{Be} + n$  destruction channel is underestimated by the large factor required to solve the  ${}^7\text{Li}$  problem.

## 5 Other possible ${}^7\text{Be}$ destruction channels

In standard BBN, the  ${}^7\text{Be}$  destruction channels involving charged nuclei are strongly sub-dominant. To produce sizeable effects on the  ${}^7\text{Li}$  abundance, their efficiency has to be increased by a very large factor. This seems possible only if new unknown resonances are found. We discuss this possibility by using the Breit-Wigner formalism.

For resonances induced by charged particle reactions at energies below the Coulomb barrier, the partial width of the entrance channel  $\Gamma_{\text{in}}$  varies very rapidly over the resonance region. That is due to the energy dependence of  $P_l(E, R)$ , see Eq.(26). In general, the partial width  $\Gamma_{\text{out}}$  of the exit channel varies more slowly since the energy of the emitted particle is increased by an amount equal to the  $Q$ -value of the reaction. We consider such a case and so neglect energy dependence of  $\Gamma_{\text{out}}$  to have a cross section form,

$$\sigma_a = \frac{\pi \omega P_l(E, R)}{2\mu E} \frac{2\xi}{\left[(E - E_r)/\gamma_{\text{in}}^2\right]^2 + [2P_l(E, R) + \xi]^2/4}, \quad (29)$$

where it has been assumed that  $\Gamma_{\text{tot}} \simeq \Gamma_{\text{in}} + \Gamma_{\text{out}}$  and  $\xi = \Gamma_{\text{out}}/\gamma_{\text{in}}^2$ . Then, for any chosen values of  $(E_r, \xi)$  and for any energy  $E$ , the cross section is an increasing function of the reduced width  $\gamma_{\text{in}}^2$ . To maximise this cross section, we assume that the reduced width of the entrance channel is equal the Wigner limiting width (Eq. (28)) that represents the maximum possible value in the approximation that the interaction potential is modelled as a square well of radius  $R$ . Moreover, we assume that the entrance channel is an  $s$ -wave ( $l = 0$ ) and that the factor  $\omega$  has the maximum value allowed by angular momentum conservation by setting  $J_{C^*} = J_a + J_{\text{Be}}$  in Eq.(25). Under these assumptions, the cross section of the resonant process is

$$\sigma_a = \frac{\pi \omega P_0(E, R)}{2\mu E} \frac{2\xi}{\left[(E - E_r)/\gamma_{\text{W}}^2(R)\right]^2 + [2P_0(E, R) + \xi]^2/4}. \quad (30)$$

It is a function of the two resonance parameters,  $(E_r, \xi)$ , and of the entrance channel radius  $R$ .

We have applied Eq. (30) to a generic  ${}^7\text{Be}$  destruction channel involving charged nuclei and then determined the effect on the  ${}^7\text{Li}$  abundance by using Eq. (14). The thermally averaged cross section  $\langle\sigma_a v\rangle_T$  has been evaluated numerically without using the narrow resonance approximation. Our results are shown in Fig. 2 as a function of the resonance parameters  $(E_r, \xi)$ . The 'coloured' lines represent the iso-contours for the  ${}^7\text{Li}$  abundance,

$$\delta Y_{\text{Li}} = 1 - \frac{Y_{\text{Li}}}{\bar{Y}_{\text{Li}}}. \quad (31)$$



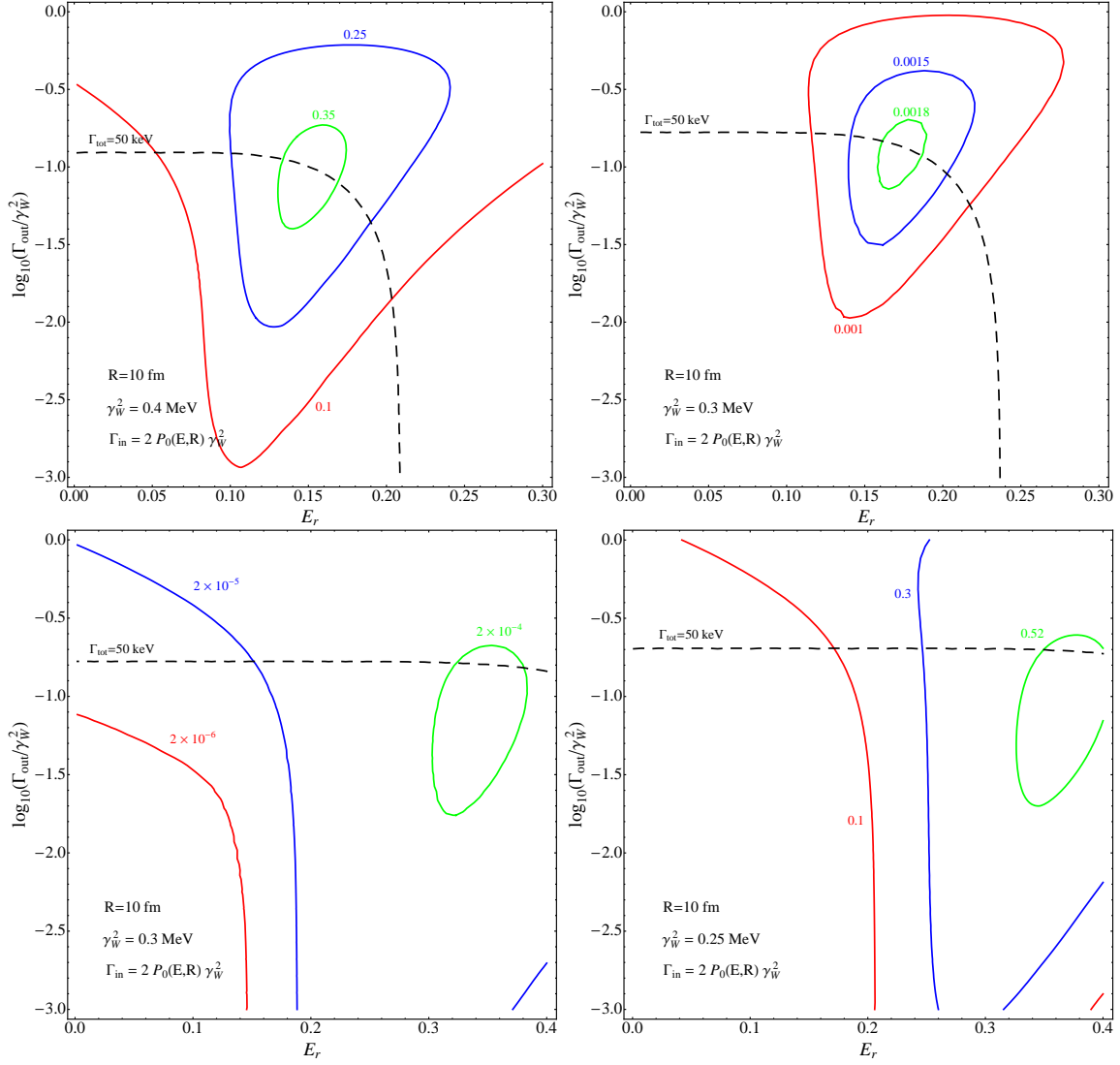


Figure 2: The coloured lines show the fractional reduction of the primordial  ${}^7\text{Li}$  abundances that can be achieved by a resonance in the  ${}^7\text{Be} + a$  reaction. The various panels correspond to  $a = d, t, {}^3\text{He}$  and  ${}^4\text{He}$ , respectively, starting from the upper left corner. The black dashed lines correspond to the condition  $\Gamma_{\text{tot}}(E_r, R) = 50$  keV, which is the limit for narrow resonance.

The various panels correspond to the processes  ${}^7\text{Be} + a$  with  $a = d, t, {}^3\text{He}$  and  ${}^4\text{He}$  respectively, starting from the upper left corner and proceeding clockwise<sup>6</sup>. In our calculations, we assumed the entrance channel radius to be  $R = 10$  fm. That is quite a large value considering the radii of the involved nuclei but it has been chosen to provide a conservative upper estimate of the resonance effects.

It is important to note that there is a maximum achievable reduction  $(\delta Y_{\text{Li}})_{\text{max}}$  of the  ${}^7\text{Li}$  abundance for each reaction channel considered; a point not discussed in previous analyses [13, 12]. Those analyses included the effects of Coulomb barrier penetration *a posteriori* and used the narrow resonance approximation to calculate  $\langle\sigma_a v\rangle_T$ . The use of the narrow resonance approximation outside its regime of validity can lead to severe over-estimation of the resonance effects. Indeed, using that assumption one has  $\langle\sigma_a v\rangle_T \sim \Gamma_{\text{eff}} (\mu T)^{-3/2} \exp(-E_r/T)$  where the effective resonance width is defined as  $\Gamma_{\text{eff}} \equiv (\Gamma_{\text{in}}\Gamma_{\text{out}})/\Gamma_{\text{tot}}$ . This expression does not predict any upper limit for  $\langle\sigma_a v\rangle_T$  as a function of  $\Gamma_{\text{eff}}$ . But a limit should exist. That can be understood by considering  $\sigma_a \leq (\pi\omega)/(2\mu E)$  for any possible choice of the resonance parameters as per Eq.(24). In fact, the correct scaling for broad resonances is given by  $\langle\sigma_a v\rangle_T \sim (\Gamma_{\text{eff}}/\Gamma_{\text{tot}}) \mu^{-1/2} T^{-3/2}$  where the factor  $\Gamma_{\text{eff}}/\Gamma_{\text{tot}} = (\Gamma_{\text{in}}\Gamma_{\text{out}})/\Gamma_{\text{tot}}^2$ , cannot be larger than one. In Fig. 2, we display with the ‘black’ dashed line the result found using the condition  $\Gamma_{\text{tot}}(E_r, R) = \Gamma_{\text{in}}(E_r, R) + \Gamma_{\text{out}} = 50$  keV. The turning point of the line corresponds to the situation  $\Gamma_{\text{in}}(E_r, R) \sim \Gamma_{\text{out}}$  and it is localised in the region where  $\xi \sim 2 P_0(E_r, R)$ . Results with a narrow resonance approximation can only be compared with those shown in the lower left corner of the plots where  $\Gamma_{\text{tot}}(E_r, R) \ll E_r$ , and the  ${}^7\text{Li}$  reduction typically is negligible.

The results that we have obtained for each specific reaction channel are now outlined sequentially:

**${}^7\text{Be} + d$ :** With this initial channel, the maximum achievable effect is a  $\sim 40\%$  reduction of primordial  ${}^7\text{Li}$  abundance. This reduction could substantially alleviate the discrepancy between theoretical predictions and observational data which differ by a factor  $\sim 3$  in the standard scenario. The maximal effect is obtained for a resonance energy  $E_r \sim 150$  keV with a total width  $\Gamma_{\text{tot}}(E_r, R) \sim 45$  keV and partial widths approximately equal to  $\Gamma_{\text{out}} \sim 35$  keV and  $\Gamma_{\text{in}}(E_r, R) \sim 10$  keV. The dependence of the maximum reduction  $(\delta Y_{\text{Li}})_{\text{max}}$  on the assumed entrance channel radius  $R$  is shown in the left panel of Fig. 3<sup>7</sup> in which  $(\delta Y_{\text{Li}})_{\text{max}}$  increases with  $R$ . That is expected given that the partial width of the entrance channel is determined primarily by Coulomb barrier penetration as in Eq. (26). So quite large values for  $R$  are needed to solve the cosmic  ${}^7\text{Li}$  problem. Even if these are much larger than the sum of the radii of the involved nuclei, 2.65 fm and 2.14 fm for  ${}^7\text{Be}$  and the deuteron respectively, they cannot be excluded in view of the large uncertainties in the approach. Our results basically coincides with those found [12] using a different approach. Note that there is a (poorly known) excited state in  ${}^9\text{B}$  at 16.71 MeV excitation. It lies just 220 keV above the  ${}^7\text{Be} + d$  threshold and it decays by gamma and particle (proton or  ${}^3\text{He}$ ) emission. The process  ${}^7\text{Be} + d \rightarrow {}^6\text{Li} + {}^3\text{He}$  could enhance  ${}^6\text{Li}$  production in the early universe. However, the effect on the *final*  ${}^6\text{Li}$  abundance is definitely too small to explain the observations of Ref.[9].

**${}^7\text{Be} + t$ :** With this initial channel, the maximum achievable effect is a  $\sim 0.2\%$  reduction of primordial  ${}^7\text{Li}$  abundance. The existence of a resonance in this channel cannot solve the cosmic  ${}^7\text{Li}$  problem, since to produce a significant  ${}^7\text{Li}$  reduction, the  ${}^7\text{Be}$  destruction rate due to the  ${}^7\text{Be} + t$  reaction should be comparable to that from  ${}^7\text{Be} + n$  processes. Clearly that is impossible because:

1. neutrons are more abundant than tritons at the relevant temperature  $T_{\text{Be}} \sim 50$  keV as is seen in Fig. 1

<sup>6</sup> We do not consider the  ${}^7\text{Be} + p$  entrance channel since this is known to be subdominant, see e.g. [13], and it is well studied at low energies due to its importance for solar neutrino production.

<sup>7</sup>The resonance parameters that maximise the  ${}^7\text{Li}$  suppression are slightly dependent on the assumed radius.

2. the cross section of  ${}^7\text{Be}(n, p){}^7\text{Li}$  is close to the unitarity bound while the  ${}^7\text{Be}+t$  collisions are suppressed by Coulomb repulsion.

Conclusions reached by others [13] differ from ours. Theirs are artifacts from using of the narrow resonance approximation outside of its regime of application.

**${}^7\text{Be} + {}^3\text{He}$ :** With this channel, the maximum achievable effect is a  $\sim 10^{-4}$  reduction in the abundance of primordial  ${}^7\text{Li}$ . Again then, a resonance in this channel cannot solve the cosmic  ${}^7\text{Li}$  problem. The small effect is due to strong Coulomb repulsion suppressing the partial width of the entrance channel ( $\Gamma_{\text{in}}(E_r, R)$ ). From our calculations, we obtain  $\Gamma_{\text{in}}(E_r, R) \sim 0.2 \text{ meV}$  and  $\Gamma_{\text{in}}(E_r, R) \sim 7 \text{ eV}$  for  $E_r \simeq 100 \text{ keV}$  and  $E_r \simeq 200 \text{ keV}$  respectively; values that are much smaller than required to obtain non negligible effects.

**${}^7\text{Be} + \alpha$ :** With this channel, the maximum achievable effect is a  $\sim 55\%$  reduction of primordial  ${}^7\text{Li}$  abundance. This is obtained for a resonance with a relatively large centroid energy  $E_r \sim 360 \text{ keV}$ , with a total width  $\Gamma_{\text{tot}}(E_r, R) \sim 21 \text{ keV}$  and with partial widths  $\Gamma_{\text{out}} \sim 19 \text{ keV}$  and  $\Gamma_{\text{in}}(E_r, R) \sim 1.5 \text{ keV}$ . The strong suppression of the cross section due to Coulomb repulsion in this case is compensated by the fact that the  $\alpha$  nuclei are  $\sim 10^6$  times more abundant than neutrons when the temperature of the universe falls below  $\sim 70 \text{ keV}$ . But this is limited to specific regions of the parameter space, only<sup>8</sup>. The dependence of the maximal achievable reduction  $(\delta Y_{\text{Li}})_{\text{max}}$  from the entrance channel radius  $R$  is explored in the right panel of Fig. 3. We see that  $R \geq 9 \text{ fm}$  is required to obtain  $(\delta Y_{\text{Li}})_{\text{max}} \geq 0.5$  and, thus, to have a substantial impact on the  ${}^7\text{Li}$  puzzle.

The possibility of a missing resonance in the last channel is particularly intriguing theoretically. We have calculated the spectrum of  ${}^{11}\text{C}$  using a coupled-channel model for the  $n$ - ${}^{10}\text{C}$  system, with coupling involving the excited quadrupole state of  ${}^{10}\text{C}$ . A multi-channel algebraic scattering (MCAS) was used with which account is made of constraints imposed by the Pauli principle on single-particle dynamics besides coupling interactions to the collective excitations of the  ${}^{10}\text{C}$  states [27]. Bound and resonant low-energy spectra of light nuclei have been analysed systematically with this approach [28, 29] and in particular for carbon isotopes [30, 31, 32]. In Table 1 a comparison is given between the calculated spectrum with the observed levels of  ${}^{11}\text{C}$  [33]. Clearly there is a one to one correspondence except for a  $\frac{1}{2}^-$  state predicted at 6.885 MeV. That excitation energy lies relatively close to entrance of  ${}^7\text{Be} + {}^4\text{He}$  channel which is 7.543 MeV above the  ${}^{11}\text{C}$  ground state and would require a  $d$ -wave collision (or a coupled-channel transition to the  ${}^7\text{Be}$  first excited state) to ensure angular momentum and parity conservation. It is worthwhile therefore to investigate this system further. To find if there is a (so far missing) resonance with about the right energy that could significantly increase cross sections.

## 6 Conclusions

We have investigated the possibility that the cosmic  ${}^7\text{Li}$  problem originates from incorrect assumptions about the nuclear reaction cross sections relevant for BBN. To do so, we introduced an efficient method to calculate the changes in the  ${}^7\text{Li}$  abundance produced by an arbitrary (temperature dependent) modification of the nuclear reaction rates. Then, taking into account that  ${}^7\text{Li}$  is mainly produced through  ${}^7\text{Be}$ , we used this method to assess whether it is possible to increase the total  ${}^7\text{Be}$  destruction rate to the level required to solve (or alleviate)

<sup>8</sup>In Fig. 9 of Ref. [13], it was shown that a narrow resonance at an energy  $E \leq 100 \text{ keV}$  and effective width  $\Gamma_{\text{eff}} \sim 10 \text{ meV}$  can substantially reduce the  ${}^7\text{Li}$  abundance. According to our analysis, however, this region of parameter space is unphysical since, if we take into account correctly the effects of Coulomb repulsion, we obtain  $\Gamma_{\text{in}}(E_r, R) \leq 1 \text{ meV}$  for  $E_r \leq 100 \text{ keV}$ .

$J^P$	Nuclear data	MCAS levels
$3/2^-$	0.00	0.00
$1/2^-$	2.000	2.915
$5/2^-$	4.3188	3.225
$3/2^-$	4.8042	3.303
$1/2^+$	6.3392	8.373
$7/2^-$	6.4782	5.768
$5/2^+$	6.9048	7.781
$1/2^-$	?	6.885
$3/2^+$	7.4997	11.059
$3/2^-$	8.1045	7.332
$5/2^-$	8.420	9.689
$7/2^+$	8.655	10.343
$5/2^+$	8.699	10.698
$5/2^+$	9.20	11.868
$3/2^-$	9.65	11.253
$5/2^-$	9.78	12.802
$7/2^-$	9.97	9.022

Table 1: Spectra of  $^{11}\text{C}$ . The data are taken from Ref. [33] while calculated values have been obtained with the MCAS formalism [27]. Potential parameters defining the MCAS coupled-channel interactions have not been sought for the optimal reproduction of levels, but only to check for possible missing resonances.

the cosmic  $^7\text{Li}$  puzzle. Given present experimental and theoretical constraints, it is unlikely that the  $^7\text{Be} + n$  destruction rate is underestimated by the  $\sim 2.5$  factor required to solve the cosmic  $^7\text{Li}$  problem.

On the basis of very general nuclear physics considerations, we have shown that the only destruction channels that could have a relevant impact on the  $^7\text{Li}$  abundance are  $^7\text{Be} + d$  and  $^7\text{Be} + \alpha$ . Our results suggest that it is unrealistic to consider new resonances in  $^7\text{Be} + t$  and  $^7\text{Be} + ^3\text{He}$  channels to solve the  $^7\text{Li}$  problem. However, with the other channels, new resonances must exist at specific energies and with suitable resonance widths. Postulating a resonance in the  $^7\text{Be} + d$  reaction at an energy  $E_r \sim 150$  keV, with a total width  $\Gamma_{\text{tot}}(E_r, R) \sim 45$  keV, and partial widths  $\Gamma_{\text{out}} \sim 35$  keV and  $\Gamma_{\text{in}}(E_r, R) \sim 10$  keV gave a  $\sim 40\%$  reduction in the  $^7\text{Li}$  abundance. A larger suppression of  $\sim 55\%$  was obtained by assuming a resonance in the  $^7\text{Be} + ^4\text{He}$  channel with energy  $E_r \sim 360$  keV, with a total width  $\Gamma_{\text{tot}}(E_r, R) \sim 21$  keV and partial widths  $\Gamma_{\text{out}} \sim 19$  keV and  $\Gamma_{\text{in}}(E_r, R) \sim 1.5$  keV. These results are the maximal achievable reductions of the  $^7\text{Li}$  abundance, since they were obtained assuming that the resonance width of the entrance channel has the largest value allowed in the presence of Coulomb repulsion, and by scanning the space of the other resonance parameters. Also, we considered a relatively large value for the entrance channel radii,  $R = 10$  fm.

In summary, a nuclear physics solution of the  $^7\text{Li}$  puzzle requires very favourable conditions. Due to the importance of the problem, and before considering more exotic explanations, further experimental and theoretical analysis of the  $^7\text{Be} + d$  and  $^7\text{Be} + \alpha$  reaction channels, specifically, are necessary. Other  $^7\text{Be}$  destruction channels do not appear influential. The  $^7\text{Be} + \alpha$  reaction is of particular interest because the comparison of the observed and calculated spectra for  $^{11}\text{C}$  have one-to-one correspondence except for a predicted  $\frac{1}{2}^-$  state that lies relatively close to the energy range of interest.

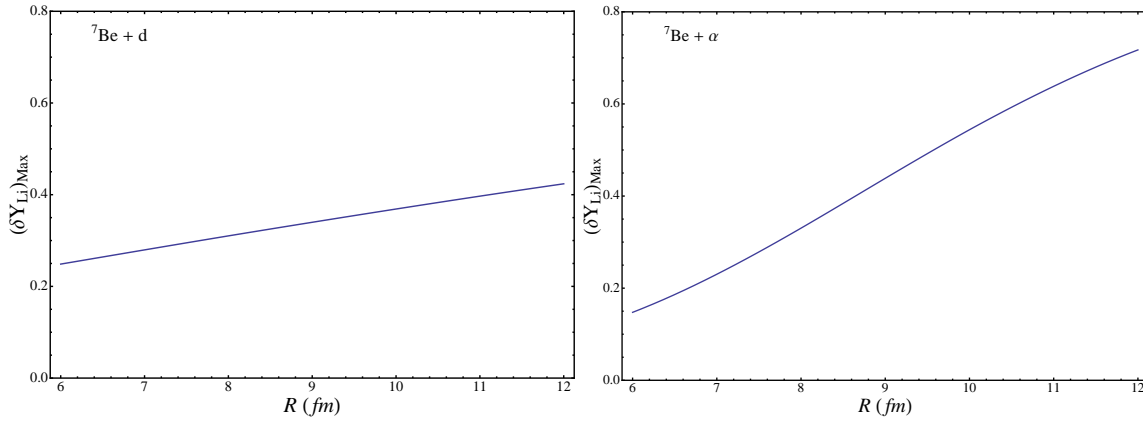


Figure 3: The maximal fractional reduction  $(\delta Y_{\text{Li}})_{\text{max}}$  of the Lithium abundance that can be achieved by a resonance in  ${}^7\text{Be} + d$  and  ${}^7\text{Be} + \alpha$  channels as a function of the assumed entrance channel radius  $R$ .

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